nag_arctanh (s11aac)

1. Purpose

nag_arctanh (s11aac) returns the value of the inverse hyperbolic tangent, $\operatorname{arctanh} x$.

2. Specification

```
#include <nag.h>
#include <nags.h>

double nag_arctanh(double x, NagError *fail)
```

3. Description

The function calculates an approximate value for the inverse hyperbolic tangent of its argument, $\operatorname{arctanh} x$.

For $x^2 \leq \frac{1}{2}$ the function is based on a Chebyshev expansion.

For $\frac{1}{2} < x^2 < 1$,

$$\operatorname{arctanh} x = \tfrac{1}{2} \ln \left(\frac{1+x}{1-x} \right).$$

4. Parameters

x

Input: the argument x of the function.
Constraint: $|\mathbf{x}| < 1.0$.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_REAL_ARG_GE

On entry, $|\mathbf{x}|$ must not be greater than or equal to 1.0: $\mathbf{x} = \langle value \rangle$.
The function has been called with an argument greater than or equal to 1.0 in magnitude, for which arctanh is not defined. The result is returned as zero.

6. Further Comments

6.1. Accuracy

If δ and ϵ are the relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{(1-x^2)\operatorname{arctanh} x}\, \delta \right|.$$

That is, the relative error in the argument, x, is amplified by at least a factor

$$\frac{x}{(1-x^2)\operatorname{arctanh} x}$$

in the result. The equality should hold if δ is greater than the ***machine precision*** (δ due to data errors etc.), but if δ is simply due to round-off in the machine representation then it is possible that an extra figure may be lost in internal calculation round-off.

The factor is not significantly greater than one except for arguments close to $|x| = 1$. However, in the region where $|x|$ is close to one, $1 - |x| \sim \delta$, the above analysis is inapplicable since x is bounded by definition, $|x| < 1$. In this region where arctanh is tending to infinity we have

$$\epsilon \sim 1/\ln \delta$$

which implies an obvious, unavoidable serious loss of accuracy near $|x| \sim 1$, e.g. if x and 1 agree to 6 significant figures, the result for $\operatorname{arctanh} x$ would be correct to at most about one figure.

6.2. References

Abramowitz M and Stegun I A (1968) *Handbook of Mathematical Functions* Dover Publications, New York ch 4.6 p 86.

7. See Also

None.

8. Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

8.1. Program Text

```
/* nag_arctanh(s11aac) Example Program
 *
 * Copyright 1989 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

main()
{
  double  x, y;

  Vprintf("s11aac Example Program Results\n");
  Vscanf("%*[^\n]");    /* skip the first input line */
  Vprintf("     x           y\n");
  while (scanf("%lf", &x) != EOF)
    {
      y = s11aac(x, NAGERR_DEFAULT);
      Vprintf("%12.3e%12.3e\n", x, y);
    }
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
s11aac Example Program Data
      -0.5
       0.0
       0.5
      -0.9999
```

8.3. Program Results

```
s11aac Example Program Results
     x           y
  -5.000e-01  -5.493e-01
   0.000e+00   0.000e+00
   5.000e-01   5.493e-01
  -9.999e-01  -4.952e+00
```